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Lagrangian for the *t–J* Model Constructed from the Generators of the Supersymmetric Hubbard Algebra

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In order to describe the dynamics of the *t–J* model, two different families of first-order Lagrangians in terms of the generators of the Hubbard algebra are found. Such families correspond to different dynamical second-class constrained systems. The quantization is carried out by using the path-integral formalism. In this context the introduction of proper ghost fields is needed to render the model renormalizable. In each case the standard Feynman diagrammatics is obtained and the renormalized physical quantities are computed and analyzed. In the first case a nonperturbative large-N expansion is considered with the purpose of studying the generalized Hubbard model describing N-fold-degenerate correlated bands. In this case the $1/N$ correction to the renormalized boson propagator is computed. In the second case the perturbative Lagrangian formalism is developed and it is shown how propagators and vertices can be renormalized to each order. In particular, the renormalized ferromagnetic magnon propagator coming from our formalism is studied in details. As an example the thermal softening of the magnon frequency is computed. The antiferromagnetic case is also analyzed, and the results are confronted with previous one obtained by means of the spin-polaron theories.

KEY WORDS: *t–J* model; Hubbard operators; Lagrangian formalism.

1. INTRODUCTION

As is known the *t–J* model is at present one of the better candidates for explaining the phenomenology of High-T_c superconductivity, and it contains the main physics of doped holes on an antiferromagnetic background (Izyumov, 1997). The *t–J* model is usually studied in the framework of the slave-particle representations (Guillou and Ragoucy, 1995). Two of them, the slave-boson and the slave-fermion, are the most important and were intensively used. The first one favors the fermion dynamics, and therefore the slave-boson representation seems to be better for describing a Fermi liquid state (Baskaran *et al.*, 1987; Kotliar and Liu, 1988a).

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Instead, the slave-fermion representation seems to give a good response when the system is closed to the antiferromagnetic order (Jayaprakash *et al.*, 1989; Kane *et al.*, 1990).

Many problems concerning the superconductivity of strongly correlated systems were treated within the context of the generalized Hubbard model by using the decoupled slave-boson representation (Grilli and Kotliar, 1990; Kotliar and Liu, 1988b; Tandon *et al.*, 1999). From the discovery of unconventional superconductivity in the rare-earth-based copper oxides and in heavy-fermion materials, interesting theoretical progresses were done in this direction. In Grilli and Kotliar (1990), Kotliar and Liu (1988b), and Tandon *et al.* (1999), the generalized Hubbard model describing N-fold-degenerate correlated bands in the infinite-U limit by means of the large-N expansion was studied. Using the slave-boson technique, Fermi-liquid properties of strongly correlated systems were evaluated. Moreover, it was shown that the leading $1/N$ corrections gives rise to different superconducting instabilities depending on the band structure and the filling factor.

As is known the slave-particle models exhibit a local gauge invariance which is destroyed in the mean field approximation. This local gauge invariance has associated a first-class constraint which is difficult to handle in the path-integral formalism.

Since the Hubbard operator representation is quite natural to treat the electronic correlation effects (Izyumov, 1997), we have developed a Lagrangian formalism in which the field variables are directly the Hubbard X-operators (Foussats *et al.*, 1999, 2000). In this approach the Hubbard $\hat{X}$-operators representing the real physical excitations are treated as indivisible objects and any decoupling scheme is used.

In the *t–J* model in which spin and charge degrees of freedom are present, the Hubbard $\hat{X}$-operators verifies the graded algebra spl(2,1) given by

$$\left[\hat{X}_i^{\alpha\beta}, \hat{X}_j^{\gamma\delta}\right]_{\pm} = \delta_{ij}\left(\delta^{\beta\gamma}\hat{X}_i^{\alpha\delta} \pm \delta^{\alpha\delta}\hat{X}_i^{\gamma\beta}\right). \tag{1.1}$$

where the indices α, β, γ, δ run in the values $+$, $-$, and 0. In Eq. (1.1), the $+$ sign must be used when both operators are fermion-like, otherwise it corresponds the $-$ sign, and i, j denote the site indices.

In order to describe the dynamics of the *t–J* model the purpose is to find the family of first-order Lagrangians written in terms of fermion-like and boson-like Hubbard X-operators. The family of Lagrangians and the constraint structure of the model are determined by using the Faddeev–Jackiw (FJ) symplectic method (Faddeev and Jackiw, 1988).

The different family of Lagrangians written in terms of field variables which verify the graded commutation rules (1.1), corresponds to different initial conditions imposed on the differential equation system produced when the FJ symplectic method is implemented. Moreover, the set of constraints is also provided by the symplectic formalism and it is second-class one (Foussats *et al.*, 1999, 2000).

We have found two kind of solutions of physical interest. As shown in Foussats *et al.* (2000), in the context of the path-integral formalism, the different family of constrained Lagrangians can be mapped into the two slave-particle representations mentioned above. In particular, the family totally constrained in the boson-like Hubbard X-operators can be mapped into the slave-boson representation and it is precisely that suitable for describing Fermi-liquid properties of strongly correlated systems. The other family we have found is mapped into the slave-fermion representation (Wiegmann, 1988, 1989).

In this paper, by means of the path-integral technique, the correlation generating functional corresponding to each family is written in terms of a suitable effective Lagrangian. Later on, we study an interesting open problem from the quantum-field theory point of view as well as from the condensed matter models (Coleman *et al.*, 2001). This is the quantization of the *t–J* model by constructing the standard Feynman diagrammatics in terms of the Hubbard X-operators.

The paper is organized as follows. In Section 2, the starting point is the correlation generating functional we have found in Foussats *et al.* (2000) for the family of Lagrangian that can be mapped into the slave-boson representation. In this case the nonperturbative formalism for the generalized Hubbard model is analyzed. This is done by means of the large-N expansion in the infinite-U limit. In Section 3, by defining proper propagators and vertices, the standard Feynman diagrammtics of the model is given. In Section 4, the boson self-energy and the renormalized boson propagator are explicitly computed. From the renormalized quantities several physical properties can be evaluated and the results confronted with others previously obtained. In Section 5, we consider the correlation generating functional (Foussats *et al.*, 1999) arising from the other family of Lagrangians that can be mapped into the slave-fermion representation. Also in this case, and by means of the perturbative formalism coming from our nonpolynomial Lagrangian, the Feynman rules, and diagrammatics are found; this is done in Section 6 for the ferromagnetic configuration. Our model is checked by computing the thermal softening of the magnon energy effect. The results are contrasted with those obtained in the framework of nonlinear spin wave model. In Section 7, the antiferromagnetic configuration is studied and the results predicted by the model are contrasted with others previously obtained by means of the spin-polaron theories.

2. LAGRANGIAN FORMALISM FOR THE GENERALIZED HUBBARD MODEL

In this section, we study the solution corresponding to the Lagrangian that can be mapped into the slave-boson representation. The starting point is to consider the following first-order Lagrangian (Foussats *et al.*, 1999) written in terms of the Hubbard X-operators

$$L = a_{\alpha\beta}(X)\dot{X}^{\alpha\beta} - \mathbf{V}^{(\mathbf{0})}(X). \tag{2.1}$$

In the FJ language, the symplectic potential $V^{(0)}$ is defined by

$$\mathbf{V}^{(\mathbf{0})} = H(X) + \lambda^a \Omega_a, \tag{2.2}$$

where λ^a are appropriate Lagrange multipliers for the constraints Ω_a.

In the Eq. (2.2), $H(X)$ is the usual t–J Hamiltonian where a term depending on the chemical potential μ was added

$$H(X) = \sum_{i,j,\sigma} t_{ij} X_i^{\sigma 0} X_j^{0\sigma} + \frac{1}{2} \sum_{ij} J_{ij} (X_i^{+-} X_j^{-+} - X_i^{++} X_j^{--}) - \mu \sum_{i,\sigma} X_i^{\sigma\sigma}. \tag{2.3}$$

In Eq. (2.3), t_{ij} and J_{ij} are respectively the hopping and the effective exchange parameters between sites i and j. The indices α, β take the values 0, $+$, and$-$, and the index σ takes the values $+$ and $-$. The five Hubbard $\hat{X}$-operators $X^{\sigma\sigma'}$ and X^{00} are boson-like and the four Hubbard $\hat{X}$-operators $X^{\sigma 0}$ and $X^{0\sigma}$ are fermion-like.

Once the FJ symplectic algorithm is implemented on the first-order Lagrangian (2.1) (Faddeev and Jackiw, 1988), a particular solution of the differential equations bring the following values for the coefficients $a_{\alpha\beta}$ and the constraints Ω_a

$$a_{i0\sigma} = \frac{i}{2X_i^{00}} X_i^{\sigma 0}, \qquad a_{i\sigma 0} = \frac{i}{2X_i^{00}} X_i^{0\sigma}, \tag{2.4}$$

$$\Omega_i^{00} = X_i^{00} + X_i^{++} + X_i^{--} - 1 = 0, \tag{2.5a}$$

$$\Omega_i^{\sigma\sigma'} = X_i^{\sigma\sigma'} - \frac{X_i^{\sigma 0} X_i^{0\sigma'}}{X_i^{00}} = 0. \tag{2.5b}$$

The boson-like Lagrangian coefficients are all zero. The set of constraints (2.5) are second-class one.

Consequently the dynamics in this condition is given by the Lagrangian

$$L(X, \dot{X}) = -\frac{i}{2X_i^{00}} \sum_{i,\sigma} \left(\dot{X}_i^{0\sigma} X_i^{\sigma 0} + \dot{X}_i^{\sigma 0} X_i^{0\sigma} \right) - H(X). \tag{2.6}$$

This solution corresponds to the configuration in which the bosons are totally constrained and the dynamics is carried out only by the fermions. The partition function corresponding to this solution reads

$$Z = \int \mathcal{D} X_i^{\alpha\beta} \delta\left[X_i^{00} + X_i^{++} + X_i^{--} - 1\right] \delta \left[X_i^{\sigma\sigma'} - \frac{X_i^{\sigma 0} X_i^{0\sigma'}}{X_i^{00}} \right] (\text{sdet}\, \mathcal{M}_{AB})_i^{\frac{1}{2}}$$
$$\times \exp\left(i \int dt\, L(X, \dot{X}) \right), \tag{2.7}$$

where

$$(\text{sdet}\, \mathcal{M}_{AB})^{\frac{1}{2}} = \left(X_i^{00}\right)^2. \tag{2.8}$$

As shown in Foussats *et al.* (2000), the correlation generating functional (2.7) can be mapped into the slave-boson representation, and therefore we consider (2.7) suitable for describing Fermi-liquid properties of strongly correlated systems.

Looking at the path-integral (2.7), the four bosonic constraints (2.5b) can be explicitly used by integrating out the delta functions. Once the transformation to the Euclidean space is done, after some algebraic manipulations the effective Lagrangian can be defined and it reads

$$
\begin{aligned}
L^{\mathrm{E}}_{\mathrm{eff}}(X,\dot{X}) = &-\frac{1}{2}\sum_{i,\sigma}\frac{1}{X_i^{00}}\left(\dot{X}_i^{0\sigma}X_i^{\sigma 0}+\dot{X}_i^{\sigma 0}X_i^{0\sigma}\right)+\sum_{i,j,\sigma}(t_{ij}-\mu\delta_{ij})X_i^{\sigma 0}X_j^{0\sigma}\\
&+\frac{1}{2}\sum_{i,j}\frac{J_{ij}}{X_i^{00}X_j^{00}}\left(X_i^{+0}X_i^{0-}X_j^{-0}X_j^{0+}+X_i^{+0}X_j^{0-}X_j^{-0}X_i^{0+}\right)\\
&+\sum_i\lambda_i\left(X^{00}-\frac{\rho}{X^{00}}-1\right),
\end{aligned}
\tag{2.9}
$$

where $\rho = X^{0+}X^{+0} + X^{0-}X^{-0}$.

As it is usual in field theory for deriving Feynman rules, the exponentiation of the superdeterminant of the symplectic supermatrix $\mathcal{M}_{AB}$ appearing in the path-integral (2.7) is carried out by introducing appropriate ghost superfields.

The 6×6 dimensional symplectic supermatrix $\mathcal{M}_{AB}$ takes the form

$$
\mathcal{M}_{AB} = \begin{pmatrix}
0 & 1+\frac{\rho}{(X^{00})^2} & \frac{1}{2}\frac{X^{0+}}{(X^{00})^2} & \frac{1}{2}\frac{X^{+0}}{(X^{00})^2} & \frac{1}{2}\frac{X^{0-}}{(X^{00})^2} & \frac{1}{2}\frac{X^{-0}}{(X^{00})^2}\\
-\left(1+\frac{\rho}{(X^{00})^2}\right) & 0 & -\frac{X^{0+}}{X^{00}} & \frac{X^{+0}}{X^{00}} & -\frac{X^{0-}}{X^{00}} & \frac{X^{-0}}{X^{00}}\\
-\frac{1}{2}\frac{X^{0+}}{(X^{00})^2} & \frac{X^{0+}}{(X^{00})} & 0 & -\frac{1}{X^{00}} & 0 & 0\\
-\frac{1}{2}\frac{X^{+0}}{(X^{00})^2} & -\frac{X^{+0}}{X^{00}} & -\frac{1}{X^{00}} & 0 & 0 & 0\\
-\frac{1}{2}\frac{X^{0-}}{(X^{00})^2} & \frac{X^{0-}}{X^{00}} & 0 & 0 & 0 & -\frac{1}{X^{00}}\\
-\frac{1}{2}\frac{X^{-0}}{(X^{00})^2} & -\frac{X^{-0}}{X^{00}} & 0 & 0 & -\frac{1}{X^{00}} & 0
\end{pmatrix},
\tag{2.10}
$$

and the superdeterminant writes

$$
(\mathrm{sdet}\,\mathcal{M}_{AB})^{\frac{1}{2}} = \frac{(\det A)^{\frac{1}{2}}}{[\det(D-CA^{-1}B)]^{\frac{1}{2}}}.
\tag{2.11}
$$

In the slave-boson representation for the generalized Hubbard model describing N-fold-degenerate correlated bands (Grilli and Kotliar, 1990; Kotliar and Liu, 1988; Tandon *et al.*, 1999), the nonperturbative large-N expansion technique is used systematically. On the other hand, also the large-N expansion was used in functional theories written in terms of the X-operators (Baym and Kadanoff, 1961; Greco and Zeyher, 1996; Zeyher and Greco, 1998; Zeyher and Kulíc, 1996). As known, in the order $1/N$ the method gives different results for superconductivity (Greco and Zeyher, 1996; Zeyher and Greco, 1998).

From our Lagrangian model used in the framework of the path-integral formalism a new nonperturbative large-N expansion is proposed (Foussats and Zandron, 2001), in order to compute renormalized physical quantities to leading order in $1/N$. To describe the generalized Hubbard model by means of the path-integral formulation, the Eq. (2.7) must be firstly arranged in such way that the $N = 2$ case is strictly equivalent to the $U = \infty$ one-band Hubbard model. We begin by relaxing the single-occupancy constraint so that a systematic loop expansion in terms of $1/N$ can be performed. Therefore, we assume that the index σ can take p values by running from 1 to N, where N is the number of electronic degrees of freedom per site, and $1/N$ can be considered as a small parameter. The symplectic supermatrix M_{AB} gets $(2 + 2N) \times (2 + 2N)$ dimension; where the Bose–Bose parts A has 2×2 dimension; the Bose–Fermi parts B has $2 \times 2N$ dimension; the Fermi–Bose parts C has $2N \times 2$ dimension and the Fermi–Fermi parts D has $2N \times 2N$ dimension.

Consequently, the boson fields in terms of statics mean-field and dynamics fluctuations are written

$$X_i^{00} = N r_0 (1 + \delta R_i), \tag{2.12a}$$

$$\lambda_i = \lambda_0 + \delta\lambda_i. \tag{2.12b}$$

Moreover the following change of variables is carried out

$$f_{ip}^{+} = \frac{1}{\sqrt{N r_o}} X_i^{p0}, \tag{2.13a}$$

$$f_{ip} = \frac{1}{\sqrt{N r_o}} X_i^{0p}. \tag{2.13b}$$

The Eqs. (2.13) show that the proportionality between the fermion field f_{ip} and the fermion-like Hubbard X_i^{0p}-operators is maintained for all orders in the large-N expansion.

Therefore, in the new variables the constraint (2.5a) takes the form

$$N r_0 (1 + \delta R_i) + \sum_p \frac{f_{ip}^{+} f_{ip}}{(1 + \delta R_i)} - \frac{N}{2} = 0, \tag{2.14}$$

where for $N = 2$ the expression (2.5a) is recovered.

The symplectic supermatrix $\mathcal{M}_{AB}$ reads

$$\mathcal{M}_{AB} = \begin{pmatrix} 0 & Nr_0 - \sum_p \frac{f_p^+ f_p}{(1+\delta R)^2} & \frac{1}{2}\frac{f_p}{(1+\delta R)^2} & \frac{1}{2}\frac{f_p^+}{(1+\delta R)^2} \\ -\left(Nr_0 - \sum_p \frac{f_p^+ f_p}{(1+\delta R)^2}\right) & 0 & -\frac{f_p}{1+\delta R} & \frac{f_p^+}{1+\delta R} \\ -\frac{1}{2}\frac{f_{p'}}{(1+\delta R)^2} & \frac{f_{p'}}{1+\delta R} & 0 & -\frac{1}{1+\delta R}\delta_{pp'} \\ -\frac{1}{2}\frac{f_{p'}^+}{(1+\delta R)^2} & -\frac{f_{p'}^+}{1+\delta R} & -\frac{1}{1+\delta R}\delta_{pp'} & 0 \end{pmatrix}. \tag{2.15}$$

The total generalized Euclidean Lagrangian is given by

$$L^{\mathrm{E}} = L_{\mathrm{eff}}^{\mathrm{E}} + L_{\mathrm{ghost}}, \tag{2.16}$$

where the effective Lagrangian is

$$\begin{aligned} L_{\mathrm{eff}}^{\mathrm{E}} = &-\frac{1}{2}\sum_{i,p}^{N}(\dot{f}_{ip} f_{ip}^+ + \dot{f}_{ip}^+ f_{ip})\left(\frac{1}{1+\delta R_i}\right) + r_o \sum_{i,j,p}^{N} t_{ij} f_{ip}^+ f_{jp} \\ &- (\mu - \lambda_0)\sum_{i,p} f_{ip}^+ f_{ip}\left(\frac{1}{1+\delta R_i}\right) + Nr_0 \sum_i \delta\lambda_i\, \delta R_i \\ &+ \sum_{i,p} f_{ip}^+ f_{ip}\left(\frac{1}{1+\delta R_i}\right)\delta\lambda_i + \frac{1}{2N}\sum_{i,j,p,p'} J_{ij}[1 - (\delta R_i + \delta R_j)] \\ &\times [f_{ip}^+ f_{ip'} f_{jp'}^+ f_{jp} + f_{ip}^+ f_{ip} f_{jp'} f_{jp'}^+]. \end{aligned} \tag{2.17}$$

The next step is to write the Lagrangian L_{ghost} obtained from the exponentiation of the superdeterminant of the symplectic supermatrix $\mathcal{M}_{AB}$. Using the Eq. (2.11) the $(\mathrm{sdet}\, \mathcal{M}_{AB})^{\frac{1}{2}}$ is written

$$(\mathrm{sdet}\, \mathcal{M}_{AB})^{\frac{1}{2}} = \frac{Nr_0}{\left(\frac{-1}{1+\delta R}\right)^N}. \tag{2.18}$$

When the integral representation for $(\mathrm{sdet}\, \mathcal{M}_{AB})^{\frac{1}{2}}$ is used, the numerator of (2.18) is written as a path-integral over Grassmann numbers (or ghost fields) θ

$$\exp\left(-\int_0^\beta d\tau\, \theta^\dagger N r_0 \theta\right), \tag{2.19}$$

and the denominator of (2.18) can be seen as a $N \times N$ diagonal matrix whose integral representation is given by using complex boson ghost fields $\mathcal{Z}_p$

$$\exp\left[-\int_0^\beta d\tau\, \mathcal{Z}_p^\dagger \left(\frac{-1}{1+\delta R}\right)\mathcal{Z}_p\right]. \tag{2.20}$$

From (2.19) the Gaussian integral is performed and its value contributes to the path-integral normalization factor. So, the θ ghost field does not appear in the formalism.

Consequently, the Lagrangian for the ghost fields $\mathcal{Z}_p$ is given by

$$L_{\text{ghost}}(\mathcal{Z}) = -\sum_{p} \mathcal{Z}_p^{\dagger} \left(\frac{1}{1+\delta R} \right) \mathcal{Z}_p. \tag{2.21}$$

From equation (2.11) and the expression (2.15), the alternative way is to consider the integral representation of the superdeterminant of the symplectic supermatrix $\mathcal{M}_{AB}$ in terms of the boson–boson 2×2 dimensional parts A and the fermion–fermion $2N \times 2N$ dimensional parts $(D - CA^{-1}B)$. In this case the integral representation gives rise to the following Lagrangian for the ghost fields

$$\begin{aligned} L_{\text{ghost}}(\theta) &+ L_{\text{ghost}}(\mathcal{Z}) \\ &= -\theta^{\dagger} \left(Nr_0 - \frac{\sum_p f_p^+ f_p}{(1+\delta R)^2} \right) \theta - \sum_{p,p'} \mathcal{Z}_p^{\dagger} \\ &\quad \times \left[\frac{1}{(1+\delta R)} \left(\delta_{pp'} - \frac{f_p^+ f_{p'}}{\left(Nr_0 - \frac{\sum_{p''} f_{p''}^+ f_{p''}}{(1+\delta R)^2}\right)(1+\delta R)^2} \right) \right] \mathcal{Z}_{p'} \end{aligned} \tag{2.22}$$

It is easy to show that the above two expressions for the ghost Lagrangians, (Eqs. (2.21) and (2.22)), yield the same results. For simplicity, the diagrammatics is constructed by considering the Eq. (2.21) for the ghost Lagrangian.

In summary, to take into account all the terms of order $1/N$, in Eq. (2.17) it is sufficient to retain term up to order δR_i^2. Therefore, the total Lagrangian L^{E} writes

$$\begin{aligned} L^{\text{E}} = &-\frac{1}{2} \sum_{i,p}^{N} (\dot{f}_{ip} f_{ip}^+ + \dot{f}_{ip}^+ f_{ip}) \big(1 - \delta R_i + \delta R_i^2\big) + r_o \sum_{i,j,p}^{N} t_{ij} f_{ip}^+ f_{jp} \\ &- (\mu - \lambda_0) \sum_{i,p} f_{ip}^+ f_{ip} \big(1 - \delta R_i + \delta R_i^2\big) + Nr_0 \sum_i \delta\lambda_i \, \delta R_i \\ &+ \sum_{i,p} f_{ip}^+ f_{ip} \big(1 - \delta R_i + \delta R_i^2\big) \delta\lambda_i + \frac{1}{2N} \sum_{i,j,p,p'} J_{ij} [1 - (\delta R_i + \delta R_j)] \\ &\times [f_{ip}^+ f_{ip'} f_{jp'}^+ f_{jp} + f_{ip}^+ f_{ip} f_{jp'} f_{jp'}^+] - \sum_p \mathcal{Z}_p^{\dagger} \big(1 - \delta R_i + \delta R_i^2\big) \mathcal{Z}_p. \end{aligned} \tag{2.23}$$

In the next section the Feynman rules and diagrammatics arising from the Lagrangian (2.23) in the infinite-U ($J_{ij} = 0$) limit are constructed.

3. FEYNMAN RULES AND DIAGRAMMATICS

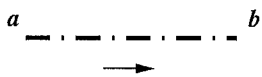
The Feynman rules can be obtained as usual. Looking at the Lagrangian (2.23), the bilinear parts give rise to the field propagators and the remaining pieces are represented by vertices. We assume the equations written in the momentum space, and so once the Fourier transformation are performed the Feynman rule propagators and vertices can be written:

(i) Propagators:

We associate with the two-component boson field $\delta X^a = (\delta R, \delta\lambda)$, the propagator

$$D_{(0)ab}(q, \omega_n) = \begin{pmatrix} 0 & \frac{1}{Nr_0} \\ \frac{1}{Nr_0} & 0 \end{pmatrix} \tag{3.1}$$

that is represented by a dotted and dashed line connecting two generic points a and b:

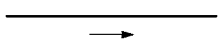


The quantities q and ω_n are the momentum and the Matsubara frequency of the bosonic field, respectively.

We associate with the N-components fermion field f_p, the propagator

$$G_{(0)pp'}(k, \nu_n) = -\frac{\delta_{pp'}}{i\nu_n - (\varepsilon_k - \mu + \lambda_0)} \tag{3.2}$$

that is represented by a line connecting two generic points p and p′:



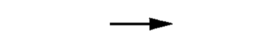
where $\varepsilon_k = -r_0 t \sum_I \exp(-i\mathbf{I}.\mathbf{k})$; and $\mathbf{I}$ is the lattice vector.

The quantities k and ν_n are the momentum and the Matsubara frequency of the fermionic field, respectively.

We associate with the N-components ghost field $\mathcal{Z}_{p'}$, the propagator

$$\mathcal{D}_{pp'} = -\delta_{pp'} \tag{3.3}$$

that is represented by a dotted line connecting two generic points p and p′:



(ii) Vertices:

The expressions for the three-leg and four-leg different vertices are respectively

$$\Lambda_a^{pp'} = (-1)\left(\frac{i}{2}(\nu + \nu') + \mu; 1\right)\delta^{pp'}. \tag{3.4}$$

$$\Lambda_{ab}^{pp'} = (-1)\begin{pmatrix} -\frac{i}{2}(\nu + \nu') - \mu & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix}\delta^{pp'}. \tag{3.5}$$

$$\Gamma_{pp'}^{a} = (-1)(\delta_{pp'}, 0) \tag{3.6}$$

$$\Gamma_{pp'}^{ab} = (-1)\begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}\delta_{pp'}. \tag{3.7}$$

4. BOSON SELF-ENERGY AND RENORMALIZED BOSON PROPAGATOR

We begin by showing how the ghost fields take part in the formalism in order to produce the cancellation of infinities in the boson self-energy. So, in order to compute the $1/N$ correction to the boson propagator, the structure of the model is examined up to one loop. By looking at the diagrammatics it can be seen that the boson self-energy Π_{ab} is given by the sum of contributions corresponding to the following one-loop diagrams

$$\Pi_{ab}(\omega, q) = \Pi_{ab}^{(1)}(\omega, q) + \Pi_{ab}^{(2)}(\omega, q) + \Pi_{ab}^{(3)}(\omega, q) + \Pi_{ab}^{(4)}(\omega, q), \tag{4.1}$$

where

$$\Pi_{ab}^{(1)}(\omega, q) = -\frac{1}{N_s}\sum_{\nu,k,p,p',p'',p'''} \Lambda_a^{p'''p} G^{pp'}(\nu, k)\Lambda_b^{p'p''} G^{p''p'''}(\nu - \omega, k - q), \tag{4.2a}$$

$$\Pi_{ab}^{(2)}(\omega, q) = -2\frac{1}{N_s}\sum_{k,\nu,p,p'} \Lambda_{ab}^{pp'} G^{pp'}(k, \nu), \tag{4.2b}$$

$$\begin{aligned}\Pi_{RR}^{(3)}(\omega, q) &= \frac{1}{N_s}\sum_{k,\nu,p,p',p'',p'''} \Gamma_R^{p'''p}\mathcal{D}_{pp'}\Gamma_R^{p'p''}\mathcal{D}_{p''p'''} \\ &= \frac{N}{N_s}\sum_k\sum_\nu 2\left(\frac{-1}{2}\right)2\left(\frac{-1}{2}\right) = \frac{N}{N_s}\sum_\nu \mathbf{1}.\end{aligned} \tag{4.2c}$$

$$\Pi_{RR}^{(4)}(\omega, q) = 2\frac{1}{N_s}\sum_{k,\nu,p,p'} \Gamma_{RR}^{pp'}\mathcal{D}_{pp'} = 2\frac{N}{N_s}\sum_\nu(-2)\left(\frac{-1}{2}\right) = 2\frac{N}{N_s}\sum_s \mathbf{1}. \tag{4.2d}$$

The other two components $(R\lambda)$ and $(\lambda\lambda)$ of $\Pi^{(3)}_{ab}$ and $\Pi^{(4)}_{ab}$ are vanishing ones. Moreover, Eqs. (4.2c,d) shows that the self-energy (RR) component of both self-energy parts has infinite value.

On the other hand, it is easy to show that the components (RR) of the self-energy parts given in (4.2a,b) have a finite part, plus infinities that are respectively cancelled with the infinities arising from the self-energy Eqs. (4.2c,d). In this way renormalized components for the boson self-energy can be obtained

$$\Pi^{(\mathrm{Ren})}_{ab}(q,\omega_n) = \Pi^{(1)\mathrm{Ren}}_{ab}(q,\omega_n) + \Pi^{(2)\mathrm{Ren}}_{ab}(q,\omega_n), \tag{4.3}$$

where

$$\Pi^{(\mathrm{Ren})}_{RR}(q,\omega_n) = -\frac{1}{4}\frac{N}{N_s}\sum_k \left[2n_F(\varepsilon_k-\mu)(\varepsilon_{k+q}-\varepsilon_k) + (\varepsilon_{k+q}+\varepsilon_k)^2\frac{[n_F(\varepsilon_{k+q}-\mu)-n_F(\varepsilon_k-\mu)]}{-i\omega_n+\varepsilon_{k+q}-\varepsilon_k}\right], \tag{4.4a}$$

$$\Pi^{(\mathrm{Ren})}_{\lambda R}(q,\omega_n) = -\frac{N}{N_s}\frac{1}{2}\sum_k(\varepsilon_{k+q}+\varepsilon_k)\frac{[n_F(\varepsilon_{k+q}-\mu)-n_F(\varepsilon_k-\mu)]}{-i\omega_n+\varepsilon_{k+q}-\varepsilon_k}, \tag{4.4b}$$

$$\Pi^{(\mathrm{Ren})}_{\lambda\lambda}(q,\omega_n) = -\frac{2N}{N_s}\frac{1}{2}\sum_k\frac{[n_F(\varepsilon_{k+q}-\mu)-n_F(\varepsilon_k-\mu)]}{-i\omega_n+\varepsilon_{k+q}-\varepsilon_k}. \tag{4.4c}$$

From the Eqs. (4.4) it can be seen that the components of the boson self-energy vanish for $q=0$.

At this stage using the Dyson equation $(D_{ab})^{-1} = (D_{(0)ab})^{-1} - \Pi^{(\mathrm{Ren})}_{ab}$ the dressed components of the matricial boson propagator can be found. So, the renormalized components are given by

$$D^{(\mathrm{Ren})}_{RR}(q,\omega_n) = \frac{1}{(Nr_0)^2}\frac{\Pi_{\lambda\lambda}}{\left[1-2\frac{\Pi_{\lambda R}}{Nr_0}+\frac{\Pi^2_{\lambda R}}{(Nr_0)^2}-\frac{\Pi_{RR}\Pi_{\lambda\lambda}}{(Nr_0)^2}\right]}, \tag{4.5a}$$

$$D^{(\mathrm{Ren})}_{\lambda\lambda}(q,\omega_n) = \frac{1}{(Nr_0)^2}\frac{\Pi_{RR}}{\left[1-2\frac{\Pi_{\lambda R}}{Nr_0}+\frac{\Pi^2_{\lambda R}}{(Nr_0)^2}-\frac{\Pi_{RR}\Pi_{\lambda\lambda}}{(Nr_0)^2}\right]}, \tag{4.5b}$$

$$D^{(\mathrm{Ren})}_{\lambda R}(q,\omega_n) = \frac{1}{(Nr_0)^2}\frac{\Pi_{\lambda R}-Nr_0}{\left[1-2\frac{\Pi_{\lambda R}}{Nr_0}+\frac{\Pi^2_{\lambda R}}{(Nr_0)^2}-\frac{\Pi_{RR}\Pi_{\lambda\lambda}}{(Nr_0)^2}\right]}. \tag{4.5c}$$

From the Eqs. (4.5) it can be seen that the components $D^{(\mathrm{Ren})}_{\lambda R}(q,\omega_n)$ and $D^{(\mathrm{Ren})}_{\lambda\lambda}(q,\omega_n)$ have a non-correct physical ultraviolet behavior, because they are constant when $\omega_n \to \infty$ [see for instance Baskaran *et al.* (1987)]. As is known the problem originates because the path-integral must be evaluated really on a discretized imaginary time. When in the path-integral the continuum limit in τ is taken carefully (Arrigoni *et al.*, 1994), the exponentials regularization factors appear, and so these unphysical singularities can be appropriately eliminated.

The renormalized boson propagator we found is the suitable one that permits us to evaluate, for instance, the $1/N$ correction to the fermion self-energy. In our model the fermion self-energy Σ is given by the sum of contributions corresponding to the following two one-loop diagrams

$$\Sigma = \Sigma^{(1)} + \Sigma^{(2)} \tag{4.6}$$

where

$$\begin{aligned}\Sigma^{(1)} &= \frac{1^{\delta_{pp}}}{N_s} \sum_{q,\omega,p',p''} \Lambda_a^{pp'} D_{(V)}^{ab}(q,\omega) \Lambda_b^{p''p} G^{p''p'}(\nu+\omega, k+q) \\ &= -\frac{\delta_{pp}}{N_s} \sum_{q,\omega_n} \left[\left(\frac{i}{2}(2\nu+\omega_n) + \mu - \lambda_0 \right)^2 D_{(V)}^{RR}(q,\omega_n) + 2\left(\frac{i}{2}(2\nu+\omega_n) \right.\right. \\ &\quad \left.\left. + \mu - \lambda_0 \right) D_{(V)}^{\lambda R}(q,\omega_n) + D_{(V)}^{\lambda\lambda}(q,\omega_n) \right] \frac{1}{i(\nu+\omega_n) - \Delta_{k+q}}\end{aligned} \tag{4.7}$$

and

$$\begin{aligned}\Sigma^{(2)} &= \frac{1^{\delta_{pp}}}{N_s} \sum_{q,\omega} \Lambda_{ab}^{pp} D_{(V)}^{ab}(q,\omega) \\ &= \frac{1^{\delta_{pp}}}{N_s} \sum_{q,\omega} \left[\left(\frac{i}{2}(2\nu+\omega_n) + \mu - \lambda_0 \right)^2 D_{(V)}^{RR}(q,\omega_n) + D_{(V)}^{\lambda R}(q,\omega_n) \right]\end{aligned} \tag{4.8}$$

Consequently, the fermion self-energy Σ reads

$$\begin{aligned}\Sigma = &-\frac{1^{\delta_{pp}}}{N_s} \sum_{q,\omega} \left[\left(\frac{i}{2}(2\nu+\omega_n) + \mu \right) \left(-\frac{i}{2}\omega_n + \Delta_{k+q} + \mu \right) D_{(V)}^{RR}(q,\omega_n) \right. \\ &\left. + (i\nu + \Delta_{k+q} + 2(\mu - \lambda_0)) D_{(V)}^{\lambda R}(q,\omega_n) + D_{(V)}^{\lambda\lambda}(q,\omega_n) \right] \\ &\times \frac{1}{i(\nu+\omega_n) - \Delta_{k+q}}\end{aligned} \tag{4.9}$$

where

$$\Delta_{k+q} = \varepsilon_{k+q} - \mu + \lambda_0. \tag{4.10}$$

From Eq. (4.9) the $1/N$ correction to the fermion self-energy can be computed [see for instance Tandon *et al.* (1999)].

Finally we remark that the diagrammatics given above was checked by computing numerically the charge–charge and spin–spin correlation functions on the square lattice for nearest-neighbor hopping t (Foussats and Greco, submitted). The results are in agreement with previous ones arising from the slave-boson model as well as from the functional X-operators canonical approach (Gehlhoff and Zeyher, 1965; Wang, 1992).

5. PERTURBATIVE LAGRANGIAN FORMALISM FOR FERROMAGNETIC AND ANTIFERROMAGNETIC CONFIGURATIONS

As was commented above, the other family of Lagrangians that can be mapped into the slave-fermion representation is useful when the system is closed to a ferromagnetic or an antiferromagnetic configuration. In this case, starting from a nonpolynomial Lagrangian, a perturbative formalism can be developed in the framework of the path-integral.

In this case, the particular solution for the coefficients of the Lagrangian (2.1) leads to the following Euclidean Lagrangian

$$L^{\mathrm{E}} = -\frac{i}{2s}\sum_i \frac{S_{i1}\dot{S}_{i2} - S_{i2}\dot{S}_{i1}}{s + S_{i3}} + \sum_{i,\sigma} \Psi_{i\sigma}\dot{\Psi}^*_{i\sigma} + H_{t-J}, \tag{5.1}$$

and the set of second-class constraints

$$\Omega = S_1^2 + S_2^2 + S_3^2 - s^2 = 0, \tag{5.2a}$$

$$\Xi_1 = \Psi^*_-(S_1 + iS_2) - \Psi^*_+(s + S_3) = 0, \tag{5.2b}$$

$$\Xi_2 = \Psi_-(S_1 - iS_2) - \Psi_+(s + S_3) = 0. \tag{5.2c}$$

We note that Eqs. (5.1) and (5.2) are written in a new set of variables [see Foussats *et al.* (1999)].

The correlation-generating function is obtained integrating the fermionic constraints (5.2b,c) and by using the integral representation for the delta function on the nonlinear bosonic constraints (5.2a). Therefore, the partition function writes

$$Z = \int \mathcal{D}S_{i1}\,\mathcal{D}S_{i2}\,\mathcal{D}S_{i3}\,\mathcal{D}\Psi_{i-}\,\mathcal{D}\Psi^*_{i-}\,\mathcal{D}\lambda_i(\mathrm{sdet}\,\mathcal{M}_{AB})_i^{\frac{1}{2}} \times \exp\left(-\int_0^\beta d\tau\, L^{\mathrm{E}}_{\mathrm{eff}}(S,\Psi)\right), \tag{5.3}$$

where $L^{\mathrm{E}}_{\mathrm{eff}}(S,\Psi)$ is defined by

$$L^{\mathrm{E}}_{\mathrm{eff}}(S,\Psi) = -\frac{i}{2s}(1-\rho)\sum_i \frac{S_{i2}\dot{S}_{i1} - S_{i1}\dot{S}_{i2}}{s + S_{i3}} - \sum_i \lambda_i\left(S_{i1}^2 + S_{i2}^2 + S_{i3}^2 - s^2\right) - s\sum_i \frac{1}{s + S_{i3}}(\dot{\Psi}^*_{i-}\Psi_{i-} + \dot{\Psi}_{i-}\Psi^*_{i-}) + H(S,\Psi). \tag{5.4}$$

The first term in Eq. (5.4) shows the nonpolynomial structure of the kinetic part of the Lagrangian.

In Eq. (5.4) the total Hamiltonian H is defined by

$$H = H_{t-J} - 2s\mu \sum_{i,\sigma} \frac{1}{s + S_{i3}} \Psi^*_{i-} \Psi_{i-}, \tag{5.5}$$

where the Hamiltonian H_{t-J} for the t–J model is given by

$$\begin{aligned} H_{t-J} = &\sum_{i,j} t_{ij} \Psi_{i-} \Psi^*_{j-} \left[1 + \left(\frac{S_{i1} - iS_{i2}}{s + S_{i3}} \right) \left(\frac{S_{j1} + iS_{j2}}{s + S_{j3}} \right) \right] \\ &- \frac{1}{8s^2} \sum_{i,j} J_{ij} (1 - \rho_i)(1 - \rho_j)[S_{i1}S_{j1} + S_{i2}S_{j2} + S_{i3}S_{j3} - s^2], \end{aligned} \tag{5.6}$$

where $J_{ij} > 0$ for a ferromagnetic state and $J_{ij} < 0$ for an antiferromagnetic one.

The simplectic supermatrix $\mathcal{M}_{AB}$ writes

$$\mathcal{M}_{AB} = \begin{pmatrix} 0 & -\frac{1-\rho}{s(s+S_3)} & \frac{(1-\rho)S_2}{2s(s+S_3)^2} & -2S_1 & 0 & 0 \\ \frac{1-\rho}{s(s+S_3)} & 0 & -\frac{(1-\rho)S_1}{2s(s+S_3)^2} & -2S_2 & 0 & 0 \\ -\frac{(1-\rho)S_2}{2s(s+S_3)^2} & \frac{(1-\rho)S_1}{2s(s+S_3)^2} & 0 & -2S_3 & i\frac{s}{(s+S_3)^2}\Psi^*_- & i\frac{s}{(s+S_3)^2}\Psi_- \\ 2S_1 & 2S_2 & 2S_3 & 0 & 0 & 0 \\ 0 & 0 & -i\frac{s}{(s+S_3)^2}\Psi^*_- & 0 & 0 & -i\frac{2s}{s+S_3} \\ 0 & 0 & -i\frac{s}{(s+S_3)^2}\Psi_- & 0 & -i\frac{2s}{s+S_3} & 0 \end{pmatrix} \tag{5.7}$$

From now on the system fluctuating around a ferromagnetic state ($J_{ij} > 0$) or an antiferromagnetic state ($J_{ij} < 0$) is assumed. In such conditions, the components of the real vector field **S** are close to be the spin variables, and so in both cases the vector **S** is written

$$\mathbf{S} = (0, 0, s') + (\tilde{S}_1, \tilde{S}_2, \tilde{S}_3) \tag{5.8}$$

where $\tilde{S}_1$, $\tilde{S}_2$, $\tilde{S}_3$ are the fluctuations. To simplify notation hereafter the tilde over the fluctuations is omitted.

To derive Feynman rules, the next step involves the rewriting of the superdeterminant of the symplectic supermatrix $\mathcal{M}_{AB}$ appearing in the partition function Eq. (5.3) as a path-integral over Faddeev–Popov ghost superfields $(\theta_\alpha, \mathcal{Z}_i)$ $(\alpha = 1, 2, 3, 4, i = 1, 2)$, such that

$$(\text{sdet}\, \mathcal{M}_{AB}) = \frac{\det A}{\det D}, \tag{5.9}$$

where A is the Bose–Bose parts and D is the Fermi–Fermi parts of the symplectic supermatrix $\mathcal{M}_{AB}$.

From Eq. (5.7) it seems that A is a real antisymmetric 4×4 dimensional matrix. By naming $I_4(A) = 4(\det A)^{1/2}$ it can be written

$$I_4 = \int \mathcal{D}\theta_\alpha \exp\left(- \int_0^\beta d\tau\, \theta^T A \theta \right), \tag{5.10}$$

where θ_α are four real Grassmann numbers or ghost fields.

Analogously, the $(\det D)^{-1/2}$ of the 2×2 dimensional matrix D is written

$$(\det D)^{-1/2} = \int \mathcal{D}\mathcal{Z}^* \mathcal{D}\mathcal{Z} \exp\left(- \int_0^\beta d\tau\, z^* C z \right), \tag{5.11}$$

where $\mathcal{Z} = \mathcal{Z}_1 + i\mathcal{Z}_2$, $\mathcal{Z}^* = \mathcal{Z}_1 - i\mathcal{Z}_2$ are complex scalar fields, and $C = \frac{2s}{s+S_3}$ $(iC = -D_{12} = -D_{21})$.

Therefore the Lagrangian L_{ghost} for the ghost fields θ_α and $\mathcal{Z}$ is given by

$$L_{\text{ghost}} = \theta^T A \theta + \mathcal{Z}^* C \mathcal{Z}, \tag{5.12}$$

and the total Lagrangian writes

$$L = L^{\text{E}}_{\text{eff}} + L_{\text{ghost}}. \tag{5.13}$$

Once the effective Lagrangian (5.4) and the matrix elements of the two matrices A and D are written in terms of the fluctuations (5.8) the total Lagrangian (5.13) is ready to construct the diagrammatics in a perturbative way.

6. DIAGRAMMATICS AND FEYNMAN RULES—FERROMAGNETIC CONFIGURATION

In this section, we begin by analysing the ferromagnetic case ($J_{ij} > 0$). The effective Lagrangian (5.4) in terms of the fluctuations (5.8) reads

$$\begin{aligned}
L^{\text{E}}_{\text{eff}} = {}& \frac{i}{2s}(1-\rho) \sum_i \frac{S_{i1}\dot{S}_{i2} - S_{i2}\dot{S}_{i1}}{s+s'} \left[1 + \sum_{n=1} (-1)^n \left(\frac{S_{i3}}{s+s'} \right)^n \right] \\
& - \frac{s}{s+s'} \sum_i (\dot{\Psi}^*_{i-}\Psi_{i-} + \dot{\Psi}_{i-}\Psi^*_{i-}) \left[1 + \sum_{n=1} (-1)^n \left(\frac{S_{i3}}{s+s'} \right)^n \right] \\
& - \frac{2s\mu}{s+s'} \sum_i \Psi^*_{i-}\Psi_{i-} \left[1 + \sum_{n=1} (-1)^n \left(\frac{S_{i3}}{s+s'} \right)^n \right] + \sum_{i,j} t_{ij} \Psi_{i-}\Psi^*_{j-} \\
& + \frac{1}{(s+s')^2} \sum_{i,j} t_{ij} \Psi_{i-}\Psi^*_{j-} [S_{i1}S_{j1} + S_{i2}S_{j2} + i(S_{i1}S_{j2} - S_{i2}S_{j1})] \\
& \times \sum_{n,m=0} (-1)^{n+m} \left(\frac{S_{i3}}{s+s'} \right)^n \left(\frac{S_{j3}}{s+s'} \right)^m - \frac{1}{8s^2} J' \sum_{i,I} \left[S_{i1} S_{(i+I)1} \right.
\end{aligned}$$

$$+ S_{i2}S_{(i+I)2} + S_{i3}S_{(i+I)3} - S_{i1}^2 - S_{i2}^2 - S_{i3}^2\big] - 2s' \sum_i \lambda_i S_{i3}$$
$$- \sum_i \lambda_i \big[S_{i1}^2 + S_{i2}^2 + S_{i3}^2\big], \tag{6.1}$$

where in Eq. (6.1) $J' = J(1-\rho)^2$, and $\sum_I$ indicates sum over nearest-neighbor sites.

Analogously, the Lagrangian for the ghost fields takes the form

$$L_{\text{ghost}}(\theta_\alpha, \mathcal{Z}) = \theta_\alpha^T (\mathcal{G}^{\alpha\beta})^{-1}\theta_\beta + \theta_\alpha^T \Gamma_a^{\alpha\beta} V^a \theta_\beta + \frac{1}{n!}\sum_{n=2} \theta_\alpha^T \Gamma_{a_1\cdots a_n}^{\alpha\beta} V^{a_1}\cdots V^{a_n}\theta_\beta$$
$$+ \mathcal{Z}^*(\mathcal{G})^{-1}\mathcal{Z} + \frac{1}{n!}\sum_{n=1} \mathcal{Z}^*\Delta_{a_1\cdots a_n} V^{a_1}\cdots V^{a_n}\mathcal{Z}. \tag{6.2}$$

From the Eqs. (6.1) and (6.2) propagators and vertices can be defined:

(i) Propagators:

We associate with the four-component boson field $V^a = (S_1, S_2, S_3, \lambda)$, the 4×4 matricial propagator

$$D_{(0)}^{ab}(q, \omega_n)$$
$$= \begin{pmatrix} s(s+s')(1+\rho)\frac{\omega_q}{\omega_q^2+\omega_n^2} & s(s+s')\frac{\omega_n}{\omega_q^2+\omega_n^2}(1+\rho) & 0 & 0 \\ -s(s+s')\frac{\omega_n}{\omega_q^2+\omega_n^2}(1+\rho) & s(s+s')(1+\rho)\frac{\omega_q}{\omega_q^2+\omega_n^2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2s'} \\ 0 & 0 & -\frac{1}{2s'} & -\frac{(1-\rho)\omega_q}{4s'^2 s(s+s')} \end{pmatrix}, \tag{6.3}$$

that is represented by a dotted and dashed line connecting two generic points a and b:

The quantities q and ω_n are respectively the momentum and the Matsubara frequency of the bosonic field.

In Eq. (6.3)

$$\omega_q = \frac{J'z}{8s}(s+s')(1+\rho)(1-\gamma_q), \tag{6.4}$$

where z is the number of first nearest-neighbor sites; $z\gamma_q = \sum_I \exp(iq.I)$; and I is the lattice vector.

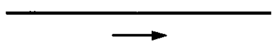
From the bosonic propagator (6.3) it is easy to obtain the free ferromagnetic magnon propagator $D_{(0)}^{-+}$ which is defined by

$$D_{(0)}^{-+} = \langle T(S^-(\tau)S^+(0))\rangle = \frac{1}{2}\left(D_{(0)}^{11} + D_{(0)}^{22} + i\left(D_{(0)}^{12} - D_{(0)}^{21}\right)\right)$$
$$= s(s+s')(1+\rho)\frac{1}{\omega_q - i\omega_n}. \tag{6.5}$$

We associate with the fermion field Ψ_- the scalar propagator

$$G_{(0)}(k,\nu_n) = \frac{s+s'}{2s}\,\frac{1}{i\nu_n + (\varepsilon_k - \mu)}, \tag{6.6}$$

that is represented by a line connecting two generic points:

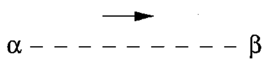


where k and ν_n are respectively the momentum and the Matsubara frequency of the fermion field, and $\varepsilon_k = -t\sum_I \exp[-ik.I]$.

We associate with the ghost field θ_α, the propagator

$$\mathcal{G}_{\alpha\beta} = (1+\rho)s(s+s')\left(\delta_\alpha^1\delta_\beta^2 - \delta_\alpha^2\delta_\beta^1\right) + \frac{1}{2s'}\left(\delta_\alpha^3\delta_\beta^4 - \delta_\alpha^4\delta_\beta^3\right), \tag{6.7}$$

that is represented by a dashed line connecting two generic points:



We associate with the ghost complex scalar field $\mathcal{Z}$, the propagator

$$\mathcal{G} = \frac{s+s'}{2s}, \tag{6.8}$$

that is represented by a dotted line:

(ii) Vertices:

The expressions of the three-leg and four-leg different vertices containing physical fields are respectively:

The three-leg boson vertex F_{abc} defined by

$$F_{abc}(\omega_1,\omega_2,\omega_3) = -\left[\frac{(1-\rho)}{2s(s+s')^2}\left[(\omega_2-\omega_1)\left(\delta_a^1\delta_b^2 - \delta_a^2\delta_b^1\right)\delta_c^3\right.\right.$$
$$+ (\omega_3-\omega_1)\left(\delta_a^1\delta_c^2 - \delta_a^2\delta_c^1\right)\delta_b^3$$
$$\left. + (\omega_3-\omega_2)\left(\delta_b^1\delta_c^2 - \delta_b^2\delta_c^1\right)\delta_a^3\right] + 2\left[\delta_a^4\left(\delta_b^1\delta_c^1 + \delta_b^2\delta_c^2\right.\right.$$

$$+ \delta_b^3\delta_c^3\big) + \delta_b^4\big(\delta_a^1\delta_c^1 + \delta_a^2\delta_c^2 + \delta_a^3\delta_c^3\big) + \delta_c^4\big(\delta_a^1\delta_b^1 + \delta_a^2\delta_b^2 + \delta_a^3\delta_b^3\big)\big]\Big]. \tag{6.9}$$

The four-leg boson vertex F_{abcd} defined by

$$\begin{aligned} F_{abcd}(\omega_1, \omega_2, \omega_3, \omega_4) = \frac{(1-\rho)}{s(s+s')^3}\big[&(\omega_2 - \omega_1)\big(\delta_a^1\delta_b^2 - \delta_a^2\delta_b^1\big)\delta_c^3\delta_d^3 \\ &+ (\omega_3 - \omega_1)\big(\delta_a^1\delta_c^2 - \delta_a^2\delta_c^1\big)\delta_b^3\delta_d^3 \\ &+ (\omega_4 - \omega_1)\big(\delta_a^1\delta_d^2 - \delta_a^2\delta_d^1\big)\delta_b^3\delta_c^3 \\ &+ (\omega_3 - \omega_2)\big(\delta_b^1\delta_c^2 - \delta_b^2\delta_c^1\big)\delta_a^3\delta_d^3 \\ &+ (\omega_4 - \omega_2)\big(\delta_b^1\delta_d^2 - \delta_b^2\delta_d^1\big)\delta_a^3\delta_c^3 \\ &+ (\omega_4 - \omega_3)\big(\delta_c^1\delta_d^2 - \delta_c^2\delta_d^1\big)\delta_a^3\delta_b^3\big]. \end{aligned} \tag{6.10}$$

The three-leg vertex K_a (two fermions-one boson) defined by

$$K_a = -\frac{s}{(s+s')^2}[i(\nu_n + \nu_n') - 2\mu]\delta_a^3. \tag{6.11}$$

The four-leg vertex K_{ab} (two fermions-two bosons) defined by

$$\begin{aligned} K_{ab} = \Bigg[&\frac{1}{(s+s')^2}\big(\varepsilon_{(k+q')} + \varepsilon_{(k'-q)}\big)\big[\delta_a^1\delta_b^1 + \delta_a^2\delta_b^2 + i\big(\delta_a^1\delta_b^2 - \delta_a^2\delta_b^1\big)\big] \\ &+ \frac{2s}{(s+s')^3}[i(\nu + \nu') - 2\mu]\delta_a^3\delta_b^3\Bigg]. \end{aligned} \tag{6.12}$$

The remaining vertices containing more than four bosons, as well as two fermions and more than two bosons can be sistematically constructed.

Finally the vertices containing ghost fields are the following:

The three-leg vertex $\Gamma_a^{\alpha\beta}$ (one-boson, two-ghost θ) defined by

$$\Gamma_1^{\alpha\beta} = -2\big(\delta_1^\alpha\delta_4^\beta - \delta_4^\alpha\delta_1^\beta\big) - \frac{1-\rho}{2s(s+s')^2}\big(\delta_2^\alpha\delta_3^\beta - \delta_3^\alpha\delta_2^\beta\big), \tag{6.13a}$$

$$\Gamma_2^{\alpha\beta} = -2\big(\delta_2^\alpha\delta_4^\beta - \delta_4^\alpha\delta_2^\beta\big) + \frac{1-\rho}{2s(s+s')^2}\big(\delta_1^\alpha\delta_3^\beta - \delta_3^\alpha\delta_1^\beta\big), \tag{6.13b}$$

$$\Gamma_3^{\alpha\beta} = -2\big(\delta_3^\alpha\delta_4^\beta - \delta_4^\alpha\delta_3^\beta\big) + \frac{1-\rho}{s(s+s')^2}\big(\delta_1^\alpha\delta_2^\beta - \delta_2^\alpha\delta_1^\beta\big), \tag{6.13c}$$

and $\Gamma_4^{\alpha\beta} = 0$.

The three-leg vertex Δ_a (one-boson, two-ghost $\mathcal{Z}$) defined by

$$\Delta_a = -\frac{2s}{(s+s')^2}\delta_a^3. \tag{6.14}$$

The remaining vertices containing two ghost fields and more than one boson can be systematically constructed.

In this way the diagrammatics is concluded and it can be used to compute renormalized quantities such as propagators, vertices, and self-energies.

For instance, the total boson self-energy is obtained by the contributions of six one-loop diagrams constructed with only physical boson and fermions, and whose analytical expressions are respectively

$$\Pi_{ab}^{(1)}(\omega, q) = \frac{1}{2N_s}\sum_{\omega',q'} F_{adc}(\omega,\omega')D_{(0)}^{de}(\omega',q')F_{ebf}(\omega,\omega') \times D_{(0)}^{fc}(\omega'-\omega, q'-q), \tag{6.15a}$$

$$\Pi_{ab}^{(2)}(\omega, q) = \frac{1}{2N_s}\sum_{\omega',q'} F_{acb}(\omega)D_{(0)}^{cd}(0)F_{def}(\omega')D_{(0)}^{ef}(\omega',q'), \tag{6.15b}$$

$$\Pi_{ab}^{(3)}(\omega, q) = \frac{1}{2N_s}\sum_{\omega',q'} F_{acdb}(\omega,\omega')D_{(0)}^{cd}(\omega',q'), \tag{6.15c}$$

$$\Pi_{ab}^{(4)}(\nu, k) = (-1)\frac{1}{N_s}\sum_{\nu,k} K_a G_0(k,\nu_n)K_b G_0(k-q,\nu_n-\omega_n), \tag{6.15d}$$

$$\Pi_{ab}^{(5)}(\nu, k) = (-1)\frac{1}{2N_s}\sum_{\nu,k} F_{acb}(\omega)D_{(0)}^{cd}(\omega')K_d G_0(k,\nu_n), \tag{6.15e}$$

$$\Pi_{ab}^{(6)}(\nu, k) = (-1)\frac{1}{2N_s}\sum_{\nu,k} K_{ab} G_0(k,\nu_n), \tag{6.15f}$$

where N_s is the lattice number of sites.

By computing the expressions (6.15), it can be seen that the infinities (constant divergences) appearing in these equations when the sum over the Matsubara frequency is carried out are mutually cancelled with the infinities appearing in the following six one-loop diagrams constructed from the ghost fields, and whose analytical expressions are respectively

$$\Pi_{ab}^{(7)}(\omega, q) = (-1)\frac{1}{2N_s}\sum_{\omega',q'} \Gamma_a^{\beta\alpha}(\omega,\omega')\mathcal{G}_{\alpha\gamma}(\omega',q')\Gamma_b^{\gamma\delta}(\omega,\omega')\mathcal{G}^{\delta\beta} \times (\omega'-\omega, q'-q), \tag{6.16a}$$

$$\Pi^{(8)}_{ab}(\omega, q) = (-1)\frac{1}{2N_s}\sum_{\omega',q'} F_{acb}(\omega)D^{cd}_{(0)}(0)\Gamma^{\beta\alpha}_{d}(\omega')\mathcal{G}_{\alpha\beta}(\omega', q'), \quad (6.16b)$$

$$\Pi^{(9)}_{ab}(\omega, q) = (-1)\frac{1}{2N_s}\sum_{\omega',q'} \Gamma^{\beta\alpha}_{ab}(\omega, \omega')\mathcal{G}_{\alpha\beta}(\omega', q'), \quad (6.16c)$$

$$\Pi^{(10)}_{ab}(\omega, q) = \frac{1}{N_s}\sum_{\omega',q'} \Delta_a\mathcal{G}\Delta_b\mathcal{G}. \quad (6.16d)$$

$$\Pi^{(11)}_{ab}(\omega, q) = \frac{1}{2N_s}\sum_{\omega',q'} F_{acb}(\omega)D^{cd}_{(0)}(\omega')\Delta_d\mathcal{G}, \quad (6.16e)$$

$$\Pi^{(12)}_{ab}(\omega, q) = \frac{1}{2N_s}\sum_{\omega',q'} \Delta_{ab}\mathcal{G}, \quad (6.16f)$$

It is important to remark that the divergences only appear in one-loop calculations. It can be seen that in more than one-loop calculations, the diagrams containing ghosts give finite contributions to the renormalized expressions of the n-point functions.

Subsequently, by means of the Dyson equation $(\mathcal{D}^{(R)})^{-1}_{ab} = (\mathcal{D}^{(R)})^{-1}_{(0)ab} - \Pi^{(R)}_{ab}$, it is possible to find the expression for the renormalized ferromagnetic magnon propagator.

By definition, the renormalized ferromagnetic magnon propagator $D^{-+}_{(R)}$ is given by

$$D^{-+}_{(R)} = \frac{1}{2}\left(D^{11}_{(R)} + D^{22}_{(R)} + i\left(D^{12}_{(R)} - D^{21}_{(R)}\right)\right). \quad (6.17)$$

where the renormalized boson propagator components are computed from the Dyson equation.

With the aim to contrast some prediction of our model with others previous well-known results obtained for instance from the nonlinear spin wave model (Mattis, 1981), it is useful to compute the correction to the magnon energy ω_q or renormalized spin-wave energy.

The renormalized ferromagnetic magnon propagator $D^{-+}_{(R)}$ becomes

$$D^{-+}_{(R)} = \frac{1}{2}\left(D^{11}_{(R)} + D^{22}_{(R)} + i\left(D^{12}_{(R)} - D^{21}_{(R)}\right)\right) = 2s^2(1+\rho)\frac{1}{\omega_q - i\omega_n - P_q(\omega_n)}, \quad (6.18)$$

where in Eq. (6.18)

$$P_q(\omega_n) = \frac{(1+\rho)}{2s^2N_s}\left(\sum_{q'} n_B(\omega_{q'})(\omega_{q'} - \omega_{q'-q}) + i\omega_n\sum_{q'} n_B(\omega_{q'})\right), \quad (6.19)$$

without taking into account the constant total fermion energy.

In Eq. (6.19), the Bose occupation number $n_B(\omega_q)$ was introduced. Moreover, including physical requirements in the calculation of the renormalized boson self-energy Π_{ab} when the sum over Matsubara frequencies is carried out, only the single pole at $\omega_q > 0$ (see Eq. (6.5)) must be taken into account.

Now, carrying out the analytic continuation $i\omega_n = \omega + i\delta$, the thermal correction to the ferromagnetic magnon energy ω_q can be found, and is given by

$$\begin{aligned}\Delta(\omega_q) &= \lim_{\delta\to 0} \text{ Re } P_q(i\omega_n = \omega_q + i\delta)\\ &= \frac{(1+\rho)}{2s^2N_s}\sum_{q'}(\omega_{q'} - \omega_{q'-q} + \omega_q)n_B(\omega_{q'}). \end{aligned} \tag{6.20}$$

Therefore the renormalized magnon energy is given by

$$\omega_q(T) = \omega_q\left[1 - \frac{4(1+\rho)}{J'z}\sum_{q'}\omega_{q'}n_B(\omega_{q'})\right], \tag{6.21}$$

showing clearly the thermal softening of the magnon frequency.

At this stage, it is possible to contrast this result of our model with others well known previously obtained from the nonlinear spin wave model in the pure bosonic case (Mattis, 1981). The finite contributions of the one loop diagrams (6.15a,b) correspond in the scheme of the nonlinear spin wave model to the direct and exchange contributions to nonlinear magnon Hamiltonian [see for instance Mattis (1981)]. That is to say, mathematically corresponds to consider that the self-energy expressions (6.15) are analytical functions in $-\omega_q$.

Therefore the renormalized components of the boson self-energy $\Pi_R^{ab}(q, \omega_n)$ in matricial form reads

$$\Pi_{(R)}^{ab}(q,\omega_n) = \begin{pmatrix} 0 & \Pi_{(R)}^{+-} & 0 & 0\\ \Pi_{(R)}^{-+} & 0 & 0 & 0\\ 0 & 0 & \Pi_{(R)}^{33} & \Pi_{(R)}^{34}\\ 0 & 0 & \Pi_{(R)}^{43} & \Pi_{(R)}^{44}\end{pmatrix}. \tag{6.22}$$

From the Eq. (6.22) the remaining different from zero components of the renormalized boson propagator $D_{(R)}^{ab}$ can be evaluated.

As well known the damping effect is given by Im P_q. We must comment that at this perturbative order and by working with free propagators and vertices, the damping is zero. The damping effect appears when in the calculations the renormalized expression for the boson propagator is used.

Analogously, by considering the fermion self-energy the renormalized expression for the fermion propagator can be constructed. It is easy to show that up to one-loop, the nonvanishing contributions to the fermion self-energy are given by the diagrams whose analytical expressions are

$$\Sigma^{(2)}(k,\nu) = \frac{1}{N_s}\sum_{q,\omega} K_{ab} D^{ab}_{(0)} \tag{6.23a}$$

$$\Sigma^{(3)}(k,\nu) = \frac{1}{N_s}\sum_{q,\omega} K_a D^{ab}_{(0)} F_{bcd} D^{cd}_{(0)} \tag{6.23b}$$

By carrying out the summation on the Matsubara frequency, the finite expression for the fermion self-energy results

$$\Sigma(k,\nu) = \Sigma^{(2)}(k,\nu) + \Sigma^{(3)}(k,\nu) = \frac{1}{N_s}\sum_q [(1+\rho)\varepsilon_{k-q} + i\nu_n - \mu] n_B(\omega_q). \tag{6.24}$$

Once more, through the Dyson equation the renormalized fermion propagator $G(k,\nu)$ can be computed and it becomes

$$G(k,\nu) = \frac{1}{(i\nu_n - \mu)\left[1 - \frac{1}{N_s}\sum_q n_B(\omega_q)\right] + \varepsilon_k\left[1 - \frac{(1+\rho)}{N_s}\sum_q \gamma_q n_B(\omega_q)\right]}. \tag{6.25}$$

Working in a similar way, the different vertices can be dressed and the expressions for the renormalized n-point functions are found.

7. ANTIFERROMAGNETIC CONFIGURATION

In this section, it is assumed that we are close to an undoped regime where the system is an antiferromagnetic insulator. Under this condition, there is a small number of holes and it can be assumed that the hole density $\rho_i = \langle \rho_i \rangle = \text{constant}$. The constant value ρ of the hole density must be determined later by consistency, for a given value of the chemical potential μ.

As is usual in the antiferromagnetics configuration a rotation of spins on the second sublattice by 180° about the S_1 axis is performed (Mattis, 1981)

$$S_{j1} \to S_{j1},\ S_{j2} \to -S_{j2},\ S_{j3} \to -S_{j3} \quad \text{and} \quad \Psi_{j\sigma} \to \Psi_{j\bar{\sigma}}, \tag{7.1}$$

where $\sigma \to \bar{\sigma}$ implies $\pm \to \mp$.

This canonical transformation changes the antiferromagnetic configuration into a ferromagnetic one with all spins up, and so it is not necessary to distinguish between sublattices. It can be seen that the effective Lagrangian (5.4) is not invariant under such transformation, because of the noninvariance of the t–J

Hamiltonian (5.6). Therefore in this case the effective Lagrangian in terms of the fluctuations (5.8) takes the form

$$\begin{aligned}
L_{\text{eff}}^{\text{E}} = {}& \frac{i}{2s}(1-\rho)\sum_i \frac{S_{i1}\dot{S}_{i2} - S_{i2}\dot{S}_{i1}}{s+s'}\left[1+\sum_{n=1}(-1)^n\left(\frac{S_{i3}}{s+s'}\right)^n\right] \\
&- \frac{s}{s+s'}\sum_i(\dot{\Psi}^*_{i-}\Psi_{i-} + \dot{\Psi}_{i-}\Psi^*_{i-})\left[1+\sum_{n=1}(-1)^n\left(\frac{S_{i3}}{s+s'}\right)^n\right] \\
&- \frac{2s\mu}{s+s'}\sum_i \Psi^*_{i-}\Psi_{i-}\left[1+\sum_{n=1}(-1)^n\left(\frac{S_{i3}}{s+s'}\right)^n\right] \\
&+ \frac{1}{2(s+s')}\sum_{i,j} t_{ij}\Psi_{i-}\Psi^*_{j-}[S_{i1} - iS_{i2} + S_{j1} + iS_{j2} + H.c] \\
&+ \frac{1}{2(s+s')}\sum_{i,j} t_{ij}\Psi_{i-}\Psi^*_{j-}\left[(S_{i1} - iS_{i2})\left(\sum_{n=1}(-1)^n\left(\frac{S_{i3}}{s+s'}\right)^n\right)\right. \\
&\left. + (S_{j1} + iS_{j2})\left(\sum_{n=1}(-1)^n\left(\frac{S_{i3}}{s+s'}\right)^n\right) + H.c\right] \\
&- \frac{1}{8s^2}J'\sum_{i,I}\left[S_{i1}S_{(i+I)1} - S_{i2}S_{(i+I)2} - S_{i3}S_{(i+I)3} + S_{i1}^2 + S_{i2}^2 + S_{i3}^2\right] \\
&- 2s'\sum_i \lambda_i S_{i3} - \sum_i \lambda_i\left[S_{i1}^2 + S_{i2}^2 + S_{i3}^2\right],
\end{aligned} \tag{7.2}$$

where now $J' < 0$.

It is easy to show that the symplectic supermatrix $\mathcal{M}_{AB}$ does not change under the canonical rotation (7.1).

From this new effective Lagrangian the results for the antiferromagnetic case can be obtained. The expression for the free boson propagator becomes

$$\mathcal{D}^{ab}_{(0)}(q,\omega_n)$$

$$= \begin{pmatrix}
-\frac{J'z}{8}(s+s')^2\frac{(1-\gamma_q)}{\omega_n^2+\omega_q^2}(1+\rho)^2 & s(s+s')\frac{\omega_n}{\omega_n^2+\omega_q^2}(1+\rho) & 0 & 0 \\
-s(s+s')\frac{\omega_n}{\omega_n^2+\omega_q^2}(1+\rho) & -\frac{J'z}{8}(s+s')^2\frac{(1+\gamma_q)}{\omega_n^2+\omega_q^2}(1+\rho)^2 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{2s'} \\
0 & 0 & -\frac{1}{2s'} & \frac{J'z(1-\gamma_q)}{32s^2s'^2}
\end{pmatrix}, \tag{7.3}$$

where the frequency ω_q is defined by

$$\omega_q = \frac{zJ'}{8s}(s+s')(1+\rho)\sqrt{1-\gamma_q^2}. \tag{7.4}$$

Consequently, the antiferromagnetic free magnon propagator remains are defined by

$$D_{(0)}^{+-} = \left(D_{(0)}^{-+}\right)^* = \langle TS^+(\tau)S^-(0)\rangle = \frac{1}{2}\left(D_{(0)}^{11} + D_{(0)}^{22} - i\left(D_{(0)}^{12} - D_{(0)}^{21}\right)\right). \tag{7.5}$$

From the Eq. (7.3) it becomes

$$D_{(0)}^{+-} = -s(s+s')(1+\rho)\left(\frac{J'z(s+s')}{8s}(1+\rho) + i\omega_n\right)\frac{1}{\omega_q^2+\omega_n^2}. \tag{7.6}$$

As well known, the antiferromagnetic magnon propagator $D_{(0)}^{+-}$, allows to define the magnon spectral function given by

$$\begin{aligned}\mathcal{A} &\equiv -\frac{1}{\pi}\lim_{\varepsilon\to 0}\operatorname{Im} D_{(0)}^{+-}(\omega+i\varepsilon)\\ &= s(s+s')(1+\rho)[A_+(q)\delta(\omega+\omega_q) - A_-(q)\delta(\omega-\omega_q)],\end{aligned} \tag{7.7}$$

where

$$A_\pm = 1 \pm \frac{1}{\sqrt{1-\gamma_q^2}}. \tag{7.8}$$

We note that the expression (7.7) for the antiferromagnetic magnon spectral function is the generalization to that given in Manousakis (1991) and Mattis (1981), when the hole density $\rho \neq 0$. This equation really check the validity of the free propagator expression (7.6) obtained for finite values of the hole density.

On the other hand, the fermionic sector in the antiferromagnetic configuration really differs from the ferromagnetic case and it must be carefully analyzed. As we will see in this case the main problem is to give the mechanism for the fermion propagation.

From the Eq. (7.2) the bilinear fermionic part reads

$$L^{\mathrm{F}} = \sum_{k,\nu_n}\Psi_-^*(k,\nu_n)G_0^{-1}\Psi_-(k,\nu_n), \tag{7.9}$$

where we have named

$$G_0^{-1} = \frac{2s}{s+s'}(i\nu_n - \mu). \tag{7.10}$$

The inverse of this scalar function given by

$$G_0 = \frac{s+s'}{2s}\frac{1}{i\nu_n-\mu}, \tag{7.11}$$

is a (nonpropagating) function which only depends on the Matsubara frequency ν_n. So, the situation for the fermionic sector is very different from the case of the ferromagnetic free fermion propagator given in Eq. (6.6).

From Lagrangian (7.2), the boson–fermion interaction vertex for the antiferromagnetic configuration can be written as

$$L_{\text{int}}^{\text{B-F}} = \frac{1}{n!}\Psi_-^*(\nu', k')U_{a_1\cdots a_n}V^{a_1}\cdots V^{a_n}\Psi_-(\nu, k), \tag{7.12}$$

where the vertex $U_{a_1\cdots a_n}$ for one and two bosonic legs explicitly reads

$$U_a = \frac{1}{s+s'}\left[(\varepsilon(k)+\varepsilon(k'))\delta_a^1 + i(\varepsilon(k')-\varepsilon(k))\delta_a^2 - \frac{s}{s+s'}[i(\nu_n+\nu_n') - 2\mu]\delta_a^3\right], \tag{7.13a}$$

$$U_{ab} = \left[\frac{-1}{(s+s')^2}(\varepsilon_{k'}+\varepsilon_k)\left[\delta_a^1\delta_b^3 + \delta_a^3\delta_b^1\right] + \frac{i}{(s+s')^2}(\varepsilon_{k'}-\varepsilon_k)\left[\delta_a^2\delta_b^3 + \delta_a^3\delta_b^2\right] + \frac{2s}{(s+s')^3}[i(\nu+\nu') - 2\mu]\delta_a^3\delta_b^3\right]. \tag{7.13b}$$

The Eqs. (7.13) show that the boson–fermion interaction for the antiferromagnetic configuration has also a different structure with respect to the ferromagnetic one.

At this stage, it is important to contrast Eq. (7.11) obtained from the bilinear fermionic part, with others' previous results given in the literature related with the spin-polaron theories (Martinez and Horsch, 1991; Schmitt-Rink *et al.*, 1988). As in these theories our starting point was to assume an antiferromagnetic order state, this physical assumption is directly connected with the fact that the fermionic modes are not propagating. Therefore, the prescriptions for the propagation of the fermionic modes must be given.

The usual way to solve the propagation of fermions is by using the Dyson equation. As is known the Dyson theorem allows to compute the inverse of the corrected fermion propagator in terms of the free fermion propagator and the self-energy. Therefore the propagator

$$G(k, \nu_n) = \left[G_0^{-1}(\nu_n) - \Sigma(k, \nu_n)\right]^{-1} \tag{7.14}$$

can be evaluated within the self-consistent Born approximation scheme (Martinez and Horsch, 1991; Schmitt-Rink *et al.*, 1988).

On the other hand, it is easy to show that in the one-loop computation of the fermion self-energy $\Sigma(k, i\nu_n)$ only one contribution coming from the three-leg vertex U_a is significant. Because of the form of the free boson propagator (7.3), the part coming from the four-leg vertex U_{ab} vanishes.

Therefore the self-energy $\Sigma(k, i\nu_n)$ is given by

$$\Sigma(k, i\nu_n) = \frac{1}{N_s} \sum_{\omega,q} U_a\, D^{ba}_{(0)}(\omega, q) U_b\, G(\nu + \omega, k + q)$$
$$= \sum_q (f(k, q) + \omega\, g(k, q)) \sum_\omega \frac{G(\nu + \omega, k + q)}{\omega^2 + \omega_q^2}$$
$$\times\, G(\nu + \omega, k + q), \tag{7.15}$$

where

$$f(k, q) = \frac{J' z (1 + \rho)^2}{4 N_s} \left(\varepsilon_k^2 + \varepsilon_{k'}^2 - 2\gamma_q \varepsilon_k \varepsilon_{k'}\right), \tag{7.16a}$$

$$g(k, q) = \frac{-2is(1 + \rho)}{(s + s')N_s} \left(\varepsilon_{k'}^2 - \varepsilon_k^2\right). \tag{7.16b}$$

By using standard techniques the following expression for the fermionic self-energy at zero temperature is found

$$\Sigma(k, i\nu_n) = \frac{(1 + \rho)}{2N_s} t^2 z^2$$
$$\times \sum_q \frac{\left[(\operatorname{sign} \gamma_q)\gamma_k \sqrt{\left[1 - \sqrt{\left(1 - \gamma_q^2\right)}\right]} - \gamma_{k+q} \sqrt{\left[1 + \sqrt{\left(1 - \gamma_q^2\right)}\right]}\right]^2}{\sqrt{\left(1 - \gamma_q^2\right)}}$$
$$\times \frac{1}{i\nu_n - \omega_q - \mu - \Sigma(k + q, i\nu_n - \omega_q)}, \tag{7.17}$$

where the relation $\varepsilon_k = -zt\gamma_k$.

Now by defining

$$u_q = \sqrt{\left(\frac{1 + \sqrt{\left(1 - \gamma_q^2\right)}}{2\sqrt{\left(1 - \gamma_q^2\right)}}\right)}, \tag{7.18a}$$

$$v_q = -(\operatorname{sign} \gamma_q) \sqrt{\left(\frac{1 - \sqrt{\left(1 - \gamma_q^2\right)}}{2\sqrt{\left(1 - \gamma_q^2\right)}}\right)}, \tag{7.18b}$$

the Eq. (7.17) takes the final form

$$\Sigma(k, i\nu_n) = \frac{(1 + \rho)}{2N_s} t^2 z^2 \sum_q \frac{(u_q \gamma_{k+q} + v_q \gamma_k)^2}{i\nu_n - \omega_q - \mu - \Sigma(k + q, i\nu_n - \omega_q)}. \tag{7.19}$$

The expression (7.19) is useful in the strong coupling case ($t > J$). Moreover, in order to describe a metallic phase where the holes move coherently on the lattice, it is necessary to solve the self-consistent equation (7.19) numerically.

Once an appropriate self-energy function $\Sigma(k, i\nu_n)$ is obtained, the propagator $G(k, \nu)$ remains well defined, and so it is possible to compute numerically the spectral function defined by $A(k, \nu) = -\frac{1}{\pi}\lim_{\varepsilon\to 0} G(k, \nu + i\varepsilon)$.

It can be seen that the Eq. (7.19) is the generalization for finite values of holes to the equivalent equation coming from the spin-polaron theories (Martinez and Horsch, 1991; Schmitt-Rink *et al.*, 1988). In fact this is a strong proof of the correctness of our quantum procedure developed in the *t–J* model.

On the other hand, because of the interaction of three or more bosons in the antiferromagnetic case remaining invariant with respect to the ferromagnetic one, the expressions for the n-leg boson vertices are unchanged. The same thing happens with the diagrammatics containing ghost fields because the symplectic supermatrix is invariant under canonical rotation, and so the ghost Lagrangian (5.12) remains unchanged.

About the boson self-energy the situation is rather different from that given for the ferromagnetic configuration. Also in the antiferromagnetic case it is possible to obtain renormalized expressions for the boson self-energy by computing the finite one-loop contributions of the different processes. Of course, all the one-loop divergences of such diagrams are cancelled again by the ghost fields.

However, in the antiferromagnetic configuration the contributions of the three diagrams containing one fermionic loop complicate the boson self-energy expression. In this case the associated renormalized matrix $\Pi^R_{ab}(q, \omega_n)$ takes the form

$$\Pi^R_{ab}(q, \omega_n) = \begin{pmatrix} \Pi^R_{11} & \Pi^R_{12} & \Pi^R_{13} & 0 \\ -\Pi^R_{12} & \Pi^R_{22} & \Pi^R_{23} & 0 \\ \Pi^R_{13} & -\Pi^R_{23} & \Pi^R_{33} & \Pi^R_{34} \\ 0 & 0 & \Pi^R_{34} & \Pi^R_{44} \end{pmatrix}. \tag{7.20}$$

Again, the renormalised antiferromagnetic magnon propagator is obtained in terms of the matrix elements $\Pi^R_{ab}(q, \omega_n)$ by using the Dyson equation.

Finally, all the results obtained in the ferromagnetic configuration for the renormalized n-point functions can be rewritten in this case by only replacing the equation for the free propagator (6.3) with the free propagator (7.3).

In summary, from the above results it is clear that the main differences with respect to the ferromagnetic case are essentially caused by two different situations. On one hand by the different forms of the free boson propagators. Contrary to the ferromagnetic case, the antiferromagnetic magnon propagator (7.6) has two single poles. Moreover, the antiferromagnetic magnon is written in terms of the hole density ρ, then at lowest order it contains only static hole density effects.

Consequently, when the hole density ρ is exactly equal to zero such magnon must be understood as the antiferromagnetic magnon at zero doping.

On the other hand, in the antiferromagnetic configuration a priori the fermions are nonpropagating particles (see Eq. (7.11)). However the prescription for propagation can be given without ambiguity by means of the Dyson equation, and the fermionic self-energy $\Sigma(k, i\nu_n)$ must be computed numerically by using Eq. (7.19).

8. CONCLUSIONS

The path-integral formalism coming from two different first-order Lagrangians written in terms of the Hubbard operators is studied. The second-class constrained systems can be respectively mapped into the two well-known decoupled slave-particle representations, i.e, slave-boson and slave-fermion ones. In this model, the Hubbard X-operators used as field variables are the generators of the graded algebra spl(2,1). These field variables allow to describe, without any decoupling assumption, spin and charge fluctuations on the atomic lattice site. In the framework of the path-integral formalism, the correlation generating function describing the dynamics of the t–J model was analyzed in two different cases and the standard Feynman diagrammatics was constructed.

Since the correlation generating functional corresponds to a second-class constrained system, the superdeterminant of the symplectic supermatrix is field-dependent and the exponentiation of such superdeterminant is realized as usual by introducing Faddeev–Popov superghost fields in the effective Lagrangian.

By following standard techniques it can be shown that ghost fields are needed in order to cancel the divergences appearing in the one-loop computation of physical quantities. In this way, the boson and the fermion self-energies and the different vertices of the model can be renormalized.

In Sections 2 and 3, the Lagrangian family that can be mapped into the slave-boson representation was studied. In this case the nonperturbative formalism for the generalized Hubbard model by using a new large-N expansion in the infinite-U limit was given. After the digrammatics and the Feynman rules were constructed, in order to compute the $1/N$ correction to the boson propagator, the structure of the model was examined in detail up to one loop. Besides the renormalized boson propagator, we found the suitable one that permits us to evaluate the $1/N$ correction to the fermion self-energy. The diagrammatics was checked by computing numerically the charge–charge and spin–spin correlation functions on the square lattice for nearest-neighbor hopping t (Foussats and Greco, 2002). The results obtained in Foussats and Greco (2002) are in agreement with previous ones arising from the slave-boson model as well as from the functional X-operators canonical approach.

Later on, the Lagrangian family that can be mapped in the slave-fermion representation was also analyzed. From our diagrammatics correct expressions for the free boson and fermion propagators are obtained. By computing the finite values of the self-energy it was possible to obtain the renormalized ferromagnetic and antiferromagnetic magnon propagators.

Two remarkable features of our approach namely (a) the thermal softening of the ferromagnetic magnon frequency and (b) the fermionic self-energy at zero temperature in the antiferromagnetic configuration (Eq. (7.19)), are respectively connected with well-known results coming from the nonlinear spin wave model and the spin-polaron theories.

In particular the softening of the ferromagnetic magnon energy obtained from our approach is the generalization for different from zero hole density of the expression obtained by means of the nonlinear spin wave model (Mattis, 1981).

At this point it is important to remark that our model accounts for the softening effect when only one-loop computations without any vertex correction is considered. We think this fact is important because in the framework of nonlinear spin wave model, the softening of the magnon energy is obtained by including vertex corrections. Really, the vertex corrections cancel scattering processes between magnons in such a way that only the direct and the exchange channels must be considered as physical processes. In our perturbative approach, the correct physical processes are directly given to each loop order.

Moreover, other important result is that in this model the divergences appear only in the one-loop structure, so the quantities are renormalized to any perturbative order.

It can be seen that in calculations at more than one loop the diagrams containing ghosts give finite contributions to the renormalized expressions of the n-point functions.

In the antiferromagnetic configuration, a scalar nonpropagating function for the fermion field was obtained. By means of the Dyson equation, the true fermion propagator can be calculated within the self-consistent Born approximation (Martinez and Horsch, 1991; Schmitt-Rink *et al.*, 1988). By using standard techniques, the expression for the fermion self-energy at zero temperature must be determined, and the fermionic propagation mechanism in the antiferromagnetic configuration is given. It can be seen that the result is the generalization for finite values of hole density to that coming from the spin-polaron theories (Martinez and Horsch, 1991; Schmitt-Rink *et al.*, 1988).

REFERENCES

Arrigoni, E., Castellani, C., Grilli, M., Raimondi, R., and Strinati, G. (1994). *Physics Report* **241**, 291–369.

Baskaran, G., Zou Z., and Anderson, P. W. (1987). *Solid State Communications* **63**, 973.

Baym, G. and Kadanoff, L. (1961). *Physical Review B: Condensed Matter* **24**, 287.

Coleman, P., Hopkinson, J., and Pépin, C. (2001). *Physical Review B: Condensed Matter* **63**, 140411.

Faddeev, L. D. and Jackiw, R. (1988). *Physical Review Letters* **60**, 1692.

Foussats, A. and Greco, A. (2002). Large-N expansion based on the Hubbard-operator path-integral representation and its application to the $t-J$ model. *Physical Review B* **65**, 1951.

Foussats, A., Greco, A., Repetto, C., Zandron, O. P., and Zandron, O. S. (2000). *Journal of Physics A: Mathematcal and General* **33**, 5849.

Foussats, A., Greco, A., and Zandron, O. S. (1999). *Annals of Physics New York* **275**, 238.

Foussats, A. and Zandron, O. (2001). Path-integral representation and Feynman diagrammatics for the generalized Hubbard model, unpublished paper.

Gehlhoff, L. and Zeyher, R. (1965). *Physical Review B: Condensed Matter* **52**, 4635.

Grilli, M. and Kotliar, G. (1990). *Physical Review Letters* **64**, 1170.

Greco, A. and Zeyher, R. (1996). *European Physical Letters* **35**, 115.

Izyumov, A. (1997). *Physics—Uspekhi* **40**, 445.

Jayaprakash, C., Krishnamurthy, H. R., and Sarker, S. (1989). *Physical Review B: Condensed Matter* **40**, 2610.

Kane, C. L., Lee, P. A., Ng, T. K., Chakraborty, B., and Read, N. (1990). *Physical Review B: Condensed Matter* **41**, 2653.

Kotliar, G. and Liu, J. (1988a). *Physical Review B: Condensed Matter* **38**, 5142.

Kotliar, G. and Liu, J. (1988). *Physical Review B: Condensed Matter* **38**, 5142.

Le Guillou, J. C. and Ragoucy, E. (1995). *Physical Review B: Condensed Matter* **52**, 2403.

Manousakis, E. (1991). *Reviews of Modern Physics* **53**, 11.

Martinez, G. and Horsch, P. (1991). *Physical Review B: Condensed Matter* **44**, 317.

Mattis, D. C. (1981). *The theory of Magnetism I*, Springer-Verlag, New York.

Schmitt-Rink, S., Varma, C. M., and Ruckenstein, A. E. (1988). *Physical Review Letters* **60**, 2793.

Tandon, A., Wang, Z., and Kotliar, G. (1999). *Physical Review Letters* **83**, 2046.

Wang, Z. (1992). *International Journal of Modern Physics B* **6**, 155.

Wiegmann, P. B. (1988). *Physical Review Letters* **60**, 821.

Wiegmann, P. B. (1989). *Nuclear Physics B* **323**, 311.

Zeyher, R. and Greco, A. (1998). *European Physics Journal B* **6**, 473.

Zeyher, R. and Kulíc, M. (1996). *Physical Review B: Condensed Matter* **53**, 2850.